A Model of Bayesian Persuasion with Transfers

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Abstract

We analyze a persuasion game with a sender and a receiver. The receiver decides whether to take an action with unknown type. In order to persuade the receiver to take the action, the sender can produce information about the benefits of the action and offer monetary transfers to the receiver. We show that limiting monetary payments can incentivize the sender to produce more information.

Keywords: Beyesian persuasion, information production, monetary transfers

JEL Codes: D83

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1 Introduction

The recent work by Kamenica and Gentzkow (2011) presents a Bayesian persuasion model in which a sender chooses what information to generate and communicate to a receiver, who then takes an action that affects the payoffs of both players. They characterize sender-optimal information structure and derive conditions under which the sender benefits from persuasion. In their analysis, the sender is not allowed to make monetary transfers to the receiver. In many situations, however, an agent can influence a decision maker's action through both information production and monetary transfers. For example, a lobbying group can influence a politician's policy decision by providing both policy relevant information and campaign contributions. A salesman can persuade a consumer to buy a particular product by offering both product information and price discounts.

Motivated by these examples, we develop a simple model of Bayesian persuasion with transfers. A person, called Sender, wants to persuade another person, called Receiver, to take a certain action. In the initial stage of the game, Sender decides what information to generate about the benefits of the action. We model Sender's information production strategy as a design of a signal. Sender controls how *informative* the signal will be. After the signal realization is known, Sender can offer monetary transfers to Receiver to influence her decision. We first characterize Sender's optimal signal and then show that limiting monetary payments can incentivize Sender to choose a more informative signal.

2 Model

Consider a persuasion game with two players: Sender (he) and Receiver (she). Receiver decides whether to take action A. Action A is "beneficial" in state H and "harmful" in state L. We use $\omega \in \{H, L\}$ to denote the state of the world. Although ω is ex ante unknown to all players, it is common knowledge that $\Pr(\omega = H) = \mu_0 \in (0,1)$.

Sender prefers Receiver to take action A regardless of action A's type. He receives benefits of v when Receiver takes action A and receives benefits of 0 otherwise. Receiver, however, prefers to choose a "beneficial" action. She receives benefits $B > 0$ from choosing a "beneficial" action, and $-L < 0$ from taking a "harmful" action.

If Receiver does not take action, her payoff is 0. We use q_0 to denote the ex ante expected benefits from taking action A:

$$q_0 = \mu_0 B - (1-\mu_0)L. \tag{1}$$

Before Receiver decides whether to take action A, Sender can influence Receiver's decision by producing information about action A's type. Formally, Sender designs a signal $S = (S_L, S_H)$, which consists of a pair of type-dependent random variables: S_L and S_H. When $\omega = L$, S produces a realization of S_L, and when $\omega = H$, S produces a realization of S_H. We allow Sender to produce no information (which is equivalent to setting $S_L = S_H$), to produce full information (which is equivalent to choosing S_L and S_H without joint support), or anything in between full and no information.

The key difference between Kamenica and Gentzkow (2011) and our model is that we allow monetary transfers from Sender to Receiver. After the signal realization is generated, Sender can offer Receiver payment $c > 0$, which he commits to pay the receiver if she takes action A.[1] We use $a \in \{1,0\}$ to denote Receiver's action. $a = 1$ when Receiver takes action A and $a = 0$ when Receiver does not take action A. Sender's payoff u_S takes the following form:

$$u_S(a,c) = \begin{cases} v - \beta c & \text{when } a = 1 \\ 0 & \text{when } a = 0 \end{cases} \tag{2}$$

where $\beta > 0$ represents Sender's cost of making monetary payments.

Receiver's payoff u_R is

$$u_R(a,\omega,c) = \begin{cases} B + \alpha c & \text{when } a = 1 \text{ and } \omega = H \\ -L + \alpha c & \text{when } a = 1 \text{ and } \omega = L \\ 0 & \text{when } a = 0 \end{cases} \tag{3}$$

where $\alpha > 0$ represents the marginal value of monetary payments to Receiver.

The game takes place in three stages. First, Sender designs signal S. Second, the signal realization is known to both players and Sender offers monetary payment to the receiver. Third, Receiver makes a decision and payoffs are realized.

[1] The contingent payment assumption is standard in the influence game studied in Grossman and Helpman (1994) and Bennedsen and Feldmann (2006).

After observing a signal realization, Receiver updates her beliefs about action type with the Bayes rule. We use μ to denote Receiver's posterior belief that $\omega = H$ and q to denote the ex post expected benefits from taking action A:

$$q = \mu B - (1-\mu)L. \tag{4}$$

When signal S has been designed, a signal realization leads to a specific value of μ and q. But before it has been realized, signal S corresponds to a distribution of q. We use random variable Q to represent the distribution of q generated by signal S. Because Q's realization q represents the ex post expected benefits of taking action A, its support must be a subset of interval $[-L, B]$. Additionally, according to the law of total expectation, the expected value of Q must be equal to the prior, i.e. $E[Q] = q_0$. We say random variable Q is *Bayes-plausible* if it satisfies these properties.

Lemma 1 *For any Bayes-plausible random variable Q, there exists a signal S for which Q is the distribution of q. For any signal S, there exists a unique Bayes-plausible random variable Q that represents the distribution of q generated by signal S.*

Lemma 1 was adapted from Cotton and Li (2016). This result greatly simplifies our analysis. To determine Sender's optimal signal, it is sufficient to ask what is the distribution of q that maximizes Sender's expected utility.

3 Equilibrium

In this section, we use backward induction to solve the subgame perfect equilibrium of the game described in section 2.

3.1 Monetary payments

Given Receiver's ex post expected benefits of action A, and Sender's monetary payment offer, Receiver takes action A when

$$q + \alpha c \geq 0. \tag{5}$$

When $q > 0$, Receiver expects positive benefits from taking action A and thus chooses action A even if Sender offers no payment. When $q < 0$, Receiver expects negative benefits from taking action A. In equilibrium, Sender offers just enough monetary payments to Receiver to ensure the implementation of action A. When q is sufficiently small, the loss associated with action A is so large that Receiver prefers not to take action A even when Sender offers the largest payment he is willing to pay (i.e. v/β). The subgame equilibrium of the second-stage game involves

$$\begin{gathered} c=0,\ \ a=1 \text{ when } q\geq 0 \\ c=-\frac{q}{\alpha},\ \ a=1 \text{ when } -\frac{\alpha v}{\beta}\leq q<0 \\ c=\frac{v}{\beta},\ \ a=0 \text{ when } q<-\frac{\alpha v}{\beta}. \end{gathered} \tag{6}$$

3.2 Optimal information strategy

In the subgame equilibrium of the second-stage game, Sender's expected payoff is a function of q.

$$Eu_S(q)=\begin{cases} v & \text{when } q>\geq 0 \\ v+q & \text{when } -\frac{\alpha v}{\beta}\leq q<0 \\ 0 & \text{when } q<-\frac{\alpha v}{\beta} \end{cases}$$

In the first stage of the game, Sender designs Bayes-plausible random variable Q. Since $q\in[-L,B]$, we can divide our analysis into two cases. Figure 1 shows Eu_S as a function of q when $\alpha<\beta L/v$. In this case, the concave closure of Eu_S takes the following form:

$$C(q_0)=\begin{cases} v+\frac{vq_0}{L} & \text{if } q_0\in[-L,0) \\ v & \text{if } q_0\in[0,B] \end{cases} \tag{7}$$

The concave closure $C(q_0)$ is the smallest concave function that is everywhere no less than Eu_S. It is a useful construct because it gives the highest payoff that Sender can achieve at prior belief q_0.[2]

If $q_0\geq 0$, producing no information leads to $q=q_0>0$ and gives Sender the highest payoff v. Therefore, Sender's optimal strategy is choosing a completely uninformative signal.[3]

If $q_0<0$, Sender's optimal strategy is choosing a partially informative signal. Specifically, he designs a signal such that Q results in $q=-L$ with probability $-q_0/L$,

[2] Kamenica and Gentzkow (2011) provides a detailed discussion on concave closure.

[3] Sender is actually indifferent over Q that always generate positive q. We assume that Sender chooses the least informative signal when he is indifferent over multiple signals. This is consistent with idea that producing more information may involve arbitrarily small costs.

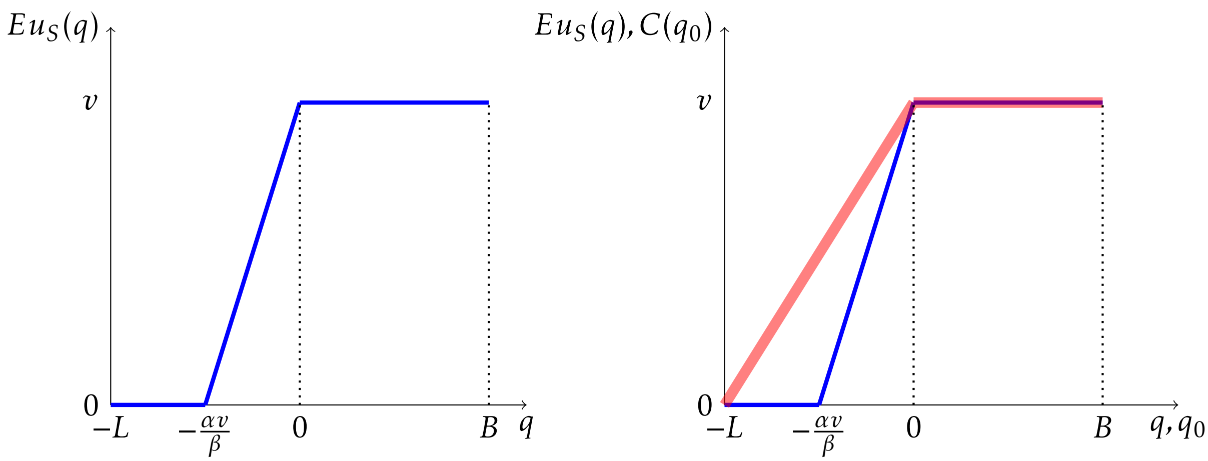

Figure 1: $Eu_S(q)$ (blue line) and its concave closure $C(q_0)$ (red line) when $\alpha < \beta L/v$.

and $q = 0$ with probability $1 + q_0/L$. Q is Bayes-plausible because $-L \times (-q_0/L) + 0 \times (1 + q_0/L) = q_0$. Q is optimal for the sender as it gives him expected payoff $0 \times (-q_0/L) + v \times (1 + q_0/L) = v + vq_0/L$, which is the highest payoff Sender can achieve (i.e. $C(q_0)$).

When $\alpha > \beta L/v$, Sender can always use monetary payments to induce the receiver to take action A. Figure 2 shows Eu_S and its concave closure when $\alpha > \beta L/v$. In figure 2, the concave closure of Eu_S coincides with Eu_S. This implies that a completely uninformative signal maximizes Sender's payoff regardless of Receiver's prior q_0.

Proposition 1 *In equilibrium, Sender's signal design is as follows:*

1. *when $q_0 \geq 0$, Sender chooses a completely uninformative signal;*

2. *when $q_0 < 0$ and $\alpha > \beta L/v$, Sender chooses a completely uninformative signal;*

3. *When $q_0 < 0$ and $\alpha < \beta L/v$, Sender chooses a partially informative signal such that Q takes the following form*

$$Q = \begin{cases} -L & \textit{with probability} \ -q_0/L \\ 0 & \textit{with probability} \ 1 + q_0/L \end{cases}$$

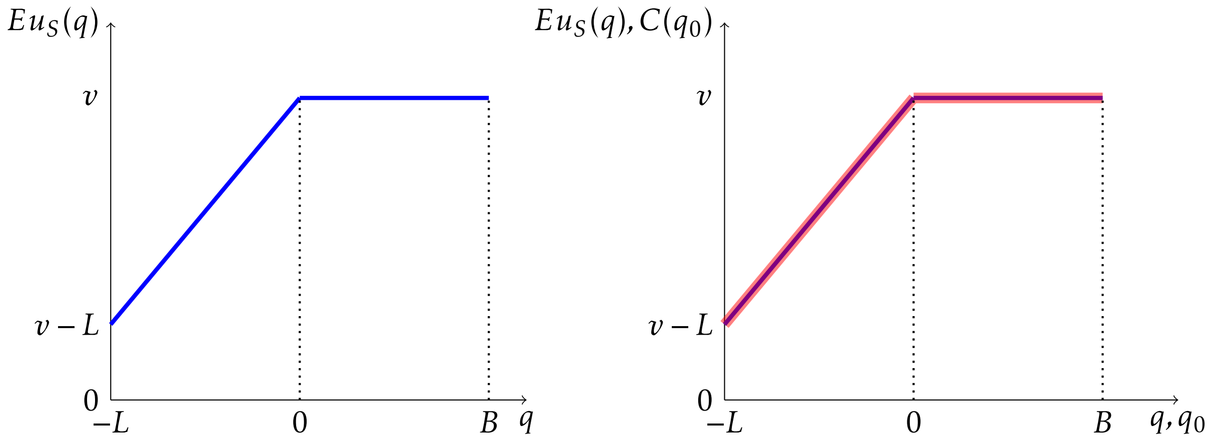

Figure 2: $Eu_S(q)$ (blue line) and its concave closure $C(q_0)$ (red line) when $\alpha > \beta L/v$.

4 Limits on monetary transfers

In this section, we consider a limit on monetary transfers, such a campaign contribution limit. We denote the limit by $\bar{c}$ and assume that $\bar{c} < v$. This changes the above analysis in that we need to replace v/β, which represents Sender's maximum willingness to pay, with $\bar{c}$, which represents Sender's maximum allowed payment.

Proposition 2 *With monetary transfers limit $\bar{c}$, Sender's signal design is as follows:*

1. *when $q_0 > 0$, Sender chooses a completely uninformative signal;*
2. *when $q_0 < 0$ and $\bar{c} > L/\alpha$, Sender chooses a completely uninformative signal;*
3. *When $q_0 < 0$ and $\bar{c} < L/\alpha$, Sender chooses a partially informative signal such that Q takes the following form*

$$Q = \begin{cases} -L & \textit{with probability} \quad -q_0/L \\ 0 & \textit{with probability} \quad 1 + q_0/L \end{cases}$$

From proposition 1, we know that if $q_0 < 0$ and $\alpha > \beta L/v$, Sender chooses a completely uninformative signal when there is no limit on monetary transfers. When this is the case, a monetary transfer limit of $\bar{c} < L/\alpha$ incentivizes Sender to produce more information: he chooses a partially informative signal under such a limit. With more information, Receiver can make a better decision regarding action A.

Proposition 3 *When $q_0 < 0$ and $\alpha > \beta L/v$, a transfer limit of $\bar{c} < L/\alpha$ incentivizes Sender to choose a more informative signal.*

5 Conclusion

We develop a simple model of Bayesian persuasion in which Sender can influence Receiver's action through both information production and monetary transfers. We show that limits on monetary tranfers may incentivize Sender to choose a more informative signal.

References

Bennedsen, Morten, and Sven E. Feldmann. 2006. "Informational Lobbying and Political Contributions." *Journal of Public Economics*, 90: 631–656.

Cotton, Christopher, and Cheng Li. 2016. "Clueless Politicians." *Queen's University Working Paper*.

Grossman, Gene M., and Elhanan Helpman. 1994. "Protection for Sale." *American Economic Review*, 84(4): 833–850.

Kamenica, Emir, and Matthew Gentzkow. 2011. "Bayesian Persuasion." *American Economic Review*, 101(6): 2590–2615.